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Singularity Resolution + Unitary Evolution + Horizon = Firewall ?

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ABSTRACT

We assume that a quantum gravity theory exists where evolutions are unitary, no information is lost, singularities are resolved, and horizons form. Thus a massive star will collapse to a black hole having a horizon and an interior singularity-resolved region. Based on unitarity and on a postulated relation, we obtain an evolution equation for the size of this region. As the black hole evolves by evaporation and accretions, this region grows, meets the horizon, and becomes accessible to an outside observer. The required time is typically of the order of black hole evaporation time. If the boundary of the singularity-resolved region can be considered as the firewall of Almheiri et al then this marks the appearance of a firewall.

(1) Consider the gravitational collapse of a very massive star. Such a collapse is believed to result in the formation of a black hole, containing a singularity at the center and a horizon surrounding it, with the constituents of the star taken to have disappeared down the singularity, and with the spacetime outside the horizon being given, for example, by the Schwarzschild solution in the static spherically symmetric case. The black hole horizon has Bekenstein–Hawking entropy and emits Hawking radiation until the horizon reaches Planckian size. As Hawking has argued, such an evolution is non unitary and leads to information loss.

In any quantum gravity theory, the evolutions are expected to be unitary, no information is expected to be lost, and the singularities are expected to be resolved. However, whether horizons are present or absent is still debatable. For a sample of works containing similar ideas regarding singularity and/or unitarity and/or horizon, see [1] – [9] where horizons are assumed to be present, [10] for Mathur’s fuzzball proposal which postulates that horizons are absent, [11] where high entropic objects are considered, and [12] where it is argued that horizons are absent.

In this paper, we assume that a quantum gravity theory exists where evolutions are unitary, no information is lost, and singularities are resolved. We further assume that horizons are present, atleast initially, having the same properties as found by standard semi classical analysis. Thus, in such a theory, the gravitational collapse of a very massive star will result in the formation of a black hole which now contains a singularity–resolved region at the center, and a horizon surrounding it which has an entropy given by Bekenstein–Hawking formula and emits Hawking radiation. The spacetime outside the horizon will be given by standard solutions in general relativity. In the following, for the sake of brevity, we refer to the singularity–resolved region at the center as singularity cloud.

We consider a spherically symmetric black hole, and analyse the consequences of the singularity resolutions and unitary evolutions on the singularity cloud and on the horizon as the black hole evolves further by evaporation and any possible accretions. Postulating a simple relation between the size of the singularity cloud and its contents, and based on the unitarity of the evolution, we obtain an equation for the size and analyse its consequences.

We find that, much before its size becomes Planckian, horizon will meet the singularity cloud. The singularity cloud then becomes accessible to an outside observer, thus the horizon ceases to exist. The boundary of this

singularity cloud can perhaps be considered as the firewall of Almheiri et al [7]. If so then access to the singularity cloud marks the appearance of firewall. The time of this appearance depends on the assumptions made about the size of the singularity cloud, and also on any accretions that may take place in the intervening period, but it is typically of the order of black hole evaporation time.

We conclude with a summary and a few questions for further studies.

(2) Consider a spherically symmetric black hole formed in a stellar collapse in $d+1$ dimensional spacetime. Let R_h, S, M , and T denote its horizon radius, entropy, mass, and Hawking temperature respectively. In Planckian units, S, M , and T may be written in terms of R_h as

$$S = C_S R_h^{d-1} \quad , \quad M = C_M R_h^{d-2} \quad , \quad T = C_T R_h^{-1} \quad (1)$$

where C_S and C_M are positive numerical constants and $C_T = \frac{d-2}{d-1} \frac{C_M}{C_S}$. Let the initial values of (R_h, S, M) be given by

$$(R_h, S, M)|_{t=0} = (R_{h0}, S_0, M_0) \gg 1 \quad (2)$$

where (R_{h0}, S_0, M_0) obey the relations given in equation (1) and $t = 0$ is taken as the black hole formation time, namely as the time when the transients of the collapse have died down and the black hole has begun to emit Hawking radiation.

The black hole evaporates by emitting Hawking radiation, therefore its mass will decrease. Also, the black hole may accrete more matter in which case its mass will increase. Thus, (R_h, S, M, T) are all functions of time. The rate of mass loss due to evaporation $\propto (Area) (temperature)^{d+1}$ and, hence, one obtains $\dot{M}_{evap} = - \frac{C_{evap}}{R_h^2}$ where C_{evap} is a positive numerical constant and an overdot denotes time derivative. Similarly, the rate of mass gain due to accretion may be written as $\dot{M}_{accr} = g(t) \geq 0$. The total rate of mass change is, therefore, given by

$$\dot{M} = \dot{M}_{evap} + \dot{M}_{accr} = - \frac{C_{evap}}{R_h^2} + g(t) \quad (3)$$

The corresponding changes in the entropy S are obtained by $\delta S = \frac{\delta M}{T}$. The total rate of entropy change is, therefore, given by

$$\dot{S} = \dot{S}_{evap} + \dot{S}_{accr} = \frac{R_h}{C_T} \left(- \frac{C_{evap}}{R_h^2} + g(t) \right) \quad (4)$$

Equations (1) and (3), or (4), lead to an equation for $R_h(t)$ given by

$$\dot{R}_h = \frac{R_h^{3-d}}{(d-2) C_M} \left(-\frac{C_{evap}}{R_h^2} + g(t) \right) , \quad (5)$$

which is to be solved with an initial condition $R_h(0) = R_{h0}$. Equations (1) then give the entropy $S(t)$ and the mass $M(t)$.

(3) Consider the interior of the black hole. In the standard semiclassical analysis, there is a singularity of Planckian size at the center and the constituents of the collapsing star are taken to have disappeared down the singularity. Now, we are assuming that a quantum gravity theory exists where the singularities are resolved, evolutions are unitary, and no information is lost. Hence, in the interior of the black hole now, there should be no singularities. Furthermore, whatever is in the interior must be capable of encoding the information about the black hole formation and its further evolution.

Let the singularity be resolved, and let the resulting singularity-resolved region be of size L and contain N number of ‘quantum gravity units’ which can encode information of order one bit each. For the sake of brevity, we refer to the singularity-resolved region as singularity cloud, and ‘quantum gravity units’ as simply units. The size L and the number of units N will depend on time t as the black hole evolves further by evaporation and accretions.

It is physically reasonable to expect, and hence we assume, that the initial number of units N_0 equals the initial entropy S_0 of the black hole formed, so that information about the collapse of the star and the formation of black hole can all be encoded in the singularity cloud. Also, in a theory where the singularities are resolved, the size L should depend on N and be parametrically much larger than Planck length. Otherwise there will be ‘remnant problems’. Thus we write $L \propto N^\sigma$ and, most conservatively, require only that $\sigma > 0$. Note that: **(i)** Semiclassically $L = \mathcal{O}(1)$, equivalently $\sigma = 0$. **(ii)** The value $\sigma = \frac{1}{d}$ is natural since it implies a density of order one unit per unit Planck volume. **(iii)** $\sigma > \frac{1}{d-1}$ implies that the initial size of the singularity cloud L_0 is larger than the initial horizon size R_{h0} which means that there is no horizon. Hence $\sigma \leq \frac{1}{d-1}$ since horizon is assumed to be present atleast initially. Thus, similarly as in equation (1), we write the relation between the size L of the singularity cloud and the number of units

N contained in it as

$$N = C_N L^\alpha \quad , \quad d-1 \leq \alpha < \infty \quad (6)$$

where C_N is a positive numerical constant and $\alpha = \frac{1}{\sigma}$. The ratio $\frac{L}{R_h}$ is given by

$$\left(\frac{L}{R_h}\right)^\alpha = \frac{C_s^{\frac{\alpha}{d-1}}}{C_N} \left(\frac{N}{S^{\frac{\alpha}{d-1}}}\right) \quad . \quad (7)$$

The assumption that horizon is present initially implies that $L_0 < R_{h0}$. Since $N_0 = S_0 \gg 1$, it then follows that $\alpha \geq d-1$ and $C_N > C_S$ if $\alpha = d-1$.

Now consider the evolution of the number N as the black hole evaporates and accretes. The number N encodes the information about evolution. Hence, the rate of change of N due to accretion should be given by

$$\dot{N}_{accr} = \dot{S}_{accr} = \frac{\dot{M}_{accr}}{T} \geq 0$$

so that accretion information can be encoded in the singularity cloud.

Consider evaporation, due to which the entropy and mass of the black hole decrease. If $\dot{N}_{evap} = \dot{S}_{evap}$ then $\dot{N}_{evap} < 0$ and, eventually, these decreasing number of units will not be able to encode the information about the formation and the evolution of the black hole. However, if $\delta N = |\delta S|$ in any process where the entropy S of the black hole is changed then $\delta N > 0$ and the information about such a change can continue to be encoded in the singularity cloud by these increasing number of units. We assume that this is the case. Then the rate of change of N due to evaporation should be given by

$$\dot{N}_{evap} = |\dot{S}_{evap}| > 0 \quad ,$$

which may also be thought of as the ingoing Hawking photons giving rise to new quantum gravity degrees of freedom upon falling into the singularity cloud. Thus, the total rate of change of the number of units N in the singularity cloud is given by

$$\dot{N} = \dot{N}_{evap} + \dot{N}_{accr} = \frac{R_h}{C_T} \left(\frac{C_{evap}}{R_h^2} + g(t) \right) > 0 \quad . \quad (8)$$

The corresponding change in the size L then follows from equation (6).

Thus, for given initial values and a given accretion rate, one solves equation (5) for $R_h(t)$. One then solves equation (8) for $N(t)$. Equations (1) and (6) then give S , M , and L as functions of t . Note that N and L increase with respect to t , and that R_h decreases if the accretion rate is sufficiently small. Thus, once the accretion rate has become sufficiently small or vanished for good, there will eventually be a time $t = t_{fw}$ when $L = R_h$. This is when the singularity cloud meets horizon and becomes accessible to an outside observer. If the boundary of the singularity cloud can be taken as the firewall of Almheiri et al then t_{fw} is the time when the firewall appears.

(4) Consider equation (5) for $R_h(t)$. It is a non autonomous, first order, ordinary differential equation. For a given arbitrary function $g(t)$, it is reasonably easy to understand the behaviour of the solution but, in general, it is not possible to obtain a closed form expression for $R_h(t)$. Hence, we first present a few illustrative examples. In the following, let t_{fw} be the time when the singularity cloud meets the horizon. Then

$$L_{fw} = R_{h\,fw} = \left(\frac{S_{fw}}{C_s} \right)^{\frac{1}{d-1}} \quad (9)$$

where the subscripts fw on the variables denote their values at $t = t_{fw}$.

Example (a) : Let there be no accretion, *i.e.* $g(t) = 0$. The solutions are then given by $N = 2S_0 - S$ and

$$R_h = R_{h0} \left(1 - \frac{t}{t_e} \right)^{\frac{1}{d}}, \quad L = L_0 \left(2 - \left(1 - \frac{t}{t_e} \right)^{\frac{d-1}{d}} \right)^{\frac{1}{\alpha}} \quad (10)$$

where the evaporation time t_e is given by

$$t_e = \frac{d-2}{d} \left(\frac{C_M}{C_{evap}} \right) R_{h0}^d \quad (11)$$

and S and M can be obtained from equations (1). Clearly R_h decreases and L increases, becoming equal at $t = t_{fw}$. Then, using $N = 2S_0 - S$, it can be seen from equation (7) that if $\alpha = d - 1$ then

$$S_{fw} = \frac{2 C_S}{C_S + C_N} S_0, \quad t_{fw} = \left(1 - \frac{2 C_S}{C_S + C_N} \right)^{\frac{d}{d-1}} t_e \quad (12)$$

whereas if $\alpha > d - 1$ then $S_{fw} \propto S_0^{\frac{d-1}{\alpha}} \ll S_0$ and $t_{fw} \sim t_e$. Also,

$$L_{fw} = R_h{}_{fw} = \left(\frac{S_{fw}}{C_s} \right)^{\frac{1}{d-1}} \sim R_{h0}^{\frac{d-1}{\alpha}} \gg 1 .$$

Example (b) : Let the accretion equal the initial evaporation, *i.e.* $g(t) = \frac{C_{evap}}{R_{h0}^2}$. The solutions are then given by $(R_h, S, M) = (R_{h0}, S_0, M_0)$ and

$$N = S_0 \left(1 + \frac{2(d-1)}{d} \frac{t}{t_e} \right) , \quad L = L_0 \left(1 + \frac{2(d-1)}{d} \frac{t}{t_e} \right)^{\frac{1}{\alpha}} \quad (13)$$

where t_e is defined in equation (11). Clearly L increases and becomes equal to R_h at $t = t_{fw}$. Then $L_{fw} = R_{h0}$ and t_{fw} is given by

$$t_{fw} = \frac{d}{2(d-1)} \left(\frac{C_N}{C_S} R_{h0}^{\alpha-(d-1)} - 1 \right) t_e \sim R_{h0}^{\alpha-(d-1)} t_e . \quad (14)$$

Thus, $t_{fw} \sim t_e$ if $\alpha = d - 1$ and t_{fw} is parametrically much larger than t_e if $\alpha > d - 1$.

Example (c) : Let there be no evaporation, *i.e.* $C_{evap} = 0$. The solutions are then given by $N = S = C_S R_h^{d-1}$ and

$$R_h = \left(\frac{M}{C_M} \right)^{\frac{1}{d-2}} , \quad M = M_0 + \int_0^t dt g(t) \quad (15)$$

where $g(t) \geq 0$. Hence, R_h , S , and M are non decreasing. Since $N = S$, we also have

$$\left(\frac{L}{R_h} \right)^\alpha = \left(\frac{L_0}{R_{h0}} \right)^\alpha \left(\frac{R_h}{R_{h0}} \right)^{d-1-\alpha} .$$

Since $L_0 < R_{h0}$ by assumption, $R_h \geq R_{h0}$, and $d - 1 \leq \alpha$, it follows that L remains less than R_h and, hence, the singularity cloud does not meet the horizon.

General Case : Consider the general case where $g(t)$ is a non negative arbitrary function. It can be seen from equation (5) that if $g(t)$ is less than the

evaporation term $\frac{C_{evap}}{R_h^2}$ then \dot{R}_h is negative, R_h decreases, $\frac{C_{evap}}{R_h^2}$ increases, and $g(t)$ becomes even less important. The solution in such a case is of the type given in example **(a)**: R_h decreases, L increases, and the singularity cloud and the horizon meet at a time t_{fw} which is of the order of evaporation time given in equation (11). Similarly, if $g(t)$ is greater than the evaporation term $\frac{C_{evap}}{R_h^2}$ then \dot{R}_h is positive, R_h increases, $\frac{C_{evap}}{R_h^2}$ decreases, and $g(t)$ becomes even more important. The solution in such a case is of the type given in example **(c)**: Both R_h and L increase, but the singularity cloud and the horizon move apart from each other and do not meet.

Note also that for a given value of R_h , the values of S and M are uniquely given by equation (1). However, unless the accretion is always absent as in example **(a)**, the values of L and N are not unique. This non uniqueness of L and N for a given value R_h can be seen in example **(b)**. In general, L and N depend on the details of the black hole evolution from the time of its formation.

Putting together these properties, one can see the following features of the solutions of equations (5), (6), and (8). They can also be proved more rigorously.

- The singularity cloud and the horizon do not meet during the accretion phase.
- They can meet only during the evaporation phase.
- The time required for this meeting is *generically* of the order of evaporation time given in equation (11), but now with R_{h0}^d replaced by $R_{h\ max}^d$ where $R_{h\ max}$ is the maximum horizon size reached due to various accretions. However, as can be seen in example **(b)**, if the model parameter $\alpha > d - 1$ then it is possible to increase this time arbitrarily by fine tuning the accretion rate.
- The black hole quantities, such as its entropy S and mass M , are uniquely fixed by its horizon radius R_h . However, the singularity cloud quantities, such as its size L and the number of units N , are not uniquely fixed by R_h but depend on the details of the black hole evolution.

In particular, knowing R_h at a given time t alone is not sufficient to predict when in future the singularity cloud will meet the horizon and become accessible to an outside observer.

The unpredictability of L and N , and hence of the time t_{fw} when the singularity cloud meets the horizon, is due to the presence of the horizon and the accretions onto the black hole. This has been referred to as ‘loss of predictability’ in our earlier works [12] and has been used to argue that horizons should be absent.

(5) In summary: We have assumed that a quantum gravity theory exists where evolutions are unitary, no information is lost, and singularities are resolved. In a gravitational collapse of a very massive star in such a theory, we further assumed that horizons are present, atleast initially, having the same semi classical properties. A black hole in such a theory has a singularity–resolved region at the center, referred to as singularity cloud, which is of size L and which contains N number of ‘quantum gravity units’ so that information about the formation and further evolution of the black hole can be encoded.

We considered a spherically symmetric black hole. Its horizon size R_h decreases due to Hawking radiation and increases due to accretions. An equation for $R_h(t)$ is easy to write. For the singularity cloud, its size L should be parametrically large and should depend on the number of units N . Postulating a simple relation between L and N , and based on the unitarity of the evolution, we then obtained an equation for $N(t)$ and thus for $L(t)$ also.

Analysing the resulting equations for R_h and N , we found that, generically as the black hole evolves, the singularity cloud increases in size, meets the horizon at time t_{fw} , and becomes accessible to an outside observer. The horizon thus ceases to exist. If the boundary of the singularity cloud can be considered as the firewall of Almheiri et al then the time t_{fw} marks the appearance of the firewall. The time t_{fw} depends on the details of the evolution but is typically of the order of black hole evaporation time.

Also, knowing R_h at a given time t is sufficient to predict the black hole quantities such as its entropy and mass. However, this alone is not sufficient to predict the singularity cloud quantities, such as its size L or the time of

access to an outsider t_{fw} , since they depend also on the details of the black hole evolution.

We now conclude by posing a few questions for further studies.

Can the boundary of the singularity cloud be taken as the firewall of Almheiri et al? Does its appearance to an outsider at time t_{fw} resolve the issues raised in their paper [7]?

When there are accretions, knowing R_h alone at a given time is not sufficient to predict t_{fw} . Since, by assumption, spacetime outside the horizon is described as in standard semiclassical analysis, there seems to be no way at all to predict t_{fw} except by knowing all the past evaporation and accretions onto the black hole since the time of its formation. Is such an unpredictability desirable, or does it point to some inconsistencies? Our own view, see [12], is that this unpredictability is undesirable, and implies that horizons do not form.

Once the singularity cloud becomes accessible to an outside observer, namely for $t > t_{fw}$, its further evolution is likely to be that of a standard quantum system with large number of interacting degrees of freedom. What is the nature of these degrees of freedom and the interactions between them? A related question is: how does the singularity cloud form in the gravitational collapse of a very massive star?

Note added : J. Pullin informed us that reference [9] contains a concrete implementation of the ideas presented here. We thank him for the information.

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